K21U 6798

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2021
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
1B01MAT : Set Theory, Differential Calculus and Numerical Methods

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark.

1. Give an example of an antisymmetric relationship.
2. Consider $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 1$. Find $f([-1, 1])$.
3. State the intermediate value theorem for continuous functions.
4. Find the domain of the real valued function $f(x, y) = \sqrt{y - x - 2}$
5. For $z = x^2y - y\cos x$, find $\frac{\partial z}{\partial x}$.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks.

6. Find the domain of the real valued function $f(x) = \sqrt{x^2 - 5x + 6}$.
7. Using arithmetic modulo M = 11, evaluate 2 – 5.
8. Give an example of a function which is not one-to-one.
9. If $\sqrt{9-2x} \le f(x) \le \sqrt{9-x^2}$ for $-1 \le x \le 1$, then find $\lim\limits_{x \to 0} f(x)$.
10. If $\lim\limits_{x\to 2}\frac{f(x)}{x^2} = 1$, find $\lim\limits_{x\to 2}\frac{f(x)}{x}$.
11. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$.
12. Show that the function $w = \sin(x + ct)$ is a solution of the wave equation $\frac{\partial^2 w}{\partial t^2} = c^2\frac{\partial^2 w}{\partial x^2}$.
13. Find $\frac{\partial z}{\partial x}$ where $yz - \ln z = x + y$ defines z as a function of x and y.

P.T.O.

14. Find all second order partial derivatives of the function $z = \frac{x}{x-y}$.

15. State Euler's theorem on homogeneous functions.

16. Determine the maximum number of positive and negative roots of the equation $3x^3 - x^2 - 10x + 1 = 0$.

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks.

17. Let ~ be a relation on Z, the set of all integers, defined by x ~ y if x – y is an integer. Is ~ an equivalence relation ? Justify your answer.

18. For $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 1$ and $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2 - 1$, find a formula for $g \circ f$. Hence or otherwise find $g \circ f(0)$.

19. Evaluate $\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$.

20. Let $f(x) = \begin{cases} -2 & x \leq -1 \\ ax - b & -1 < x < 1 \\ 3 & x \geq 1 \end{cases}$

For what value of a and b is f continuous at every x ?

21. Does the function $f(x, y) = \frac{x - y}{x + y}$ have a limit as $(x, y) \to (0, 0)$? Justify your answer.

22. Let $u = \sin^{-1} \frac{x^2 + y^2}{x + y}$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

23. Find using method of false position, a positive root of the equation $x - e^{-x} = 0$ correct to two decimal places.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks.

24. a) Let a function f be defined by $f(x) = \frac{3x + 2}{x - 1}$. Find a formula for f^{-1}.

b) Prove that $\log_b AB = \log_b A + \log_b B$.

25. If $y = \sin^{-1}x$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$. Further, find $(y_n)0$.

26. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at $\left(\frac{1}{2}, 1\right)$ where $w = xy + yz + xz$, $x = u + v$, $y = u - v$ and $z = uv$.

27. Derive the Newton's method for finding 1/N, where N > 0. Hence, find 1/17 correct to four decimal places, using the initial approximation $x_0 = 0.05$.
